Mumford dendrograms and discrete p-adic symmetries

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In this talk, we present an effective encoding of dendrograms by embedding them into the Bruhat-Tits trees associated to p-adic number fields. As an application, we show how strings over a finite alphabet can be encoded in cyclotomic extensions of $\mathbb{Q}_p$ and discuss p-adic DNA encoding. The application leads to fast p-adic agglomerative hierarchic algorithms similar to the ones recently used e.g. by A. Khrennikov and others. From the viewpoint of p-adic geometry, to encode a dendrogram X in a p-adic field K means to fix a set S of K-rational punctures on the p-adic projective line $\mathbb{P}^1$. To $\mathbb{P}^1 \setminus S$ is associated in a natural way a subtree inside the Bruhat-Tits tree which recovers X, a method first used by F. Kato in 1999 in the classification of discrete subgroups of $\mathrm{PGL}_2(K)$.

Next, we show how the p-adic moduli space $\mathfrak{M}_{0,n}$ of $\mathbb{P}^1$ with n punctures can be applied to the study of time series of dendrograms and those symmetries arising from hyperbolic actions on $\mathbb{P}^1$. In this way, we can associate to certain classes of dynamical systems a Mumford curve, i.e. a p-adic algebraic curve with totally degenerate reduction modulo p.

In the end, we indicate some of our results in the study of general discrete actions on $\mathbb{P}^1$, and their relation to p-adic Hurwitz spaces.